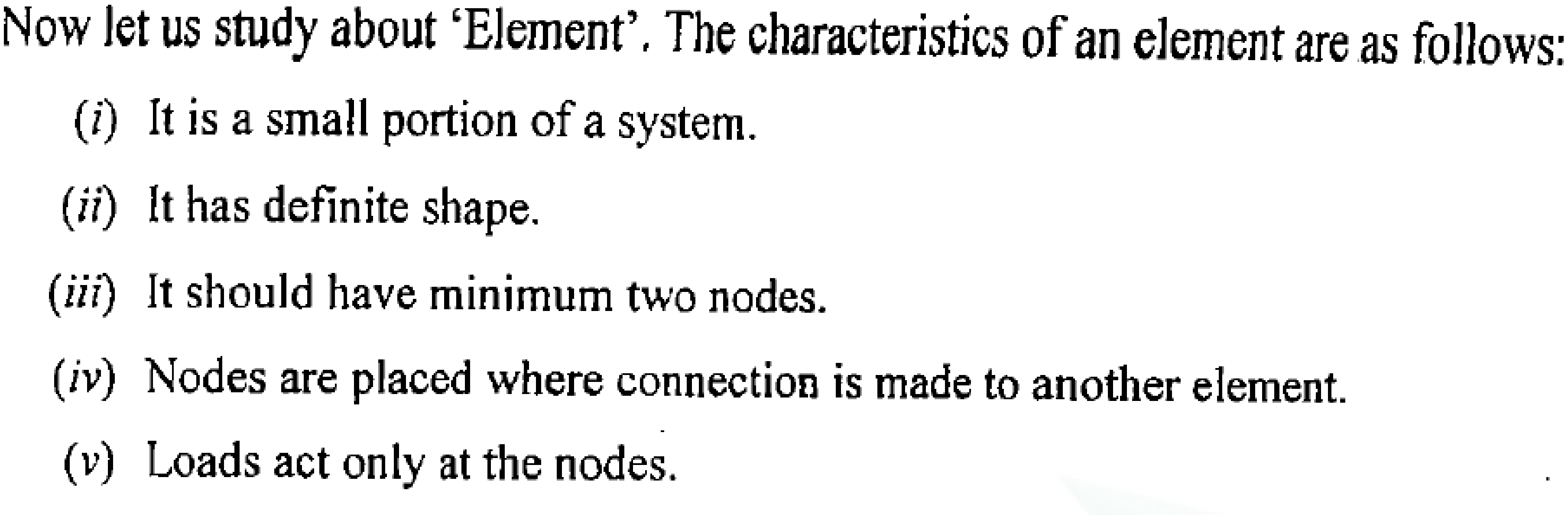
Discretization

- The art of subdividing a structure in to convenient number of smaller elements is known as Discretization



Now let us study about 'Element'. The characteristics of an element are as follows:

(*i*) It is a small portion of a system.

(*ii*) It has definite shape.

(*iii*) It should have minimum two nodes.

(*iv*) Nodes are placed where connection is made to another element.

(*v*) Loads act only at the nodes.

Examples:

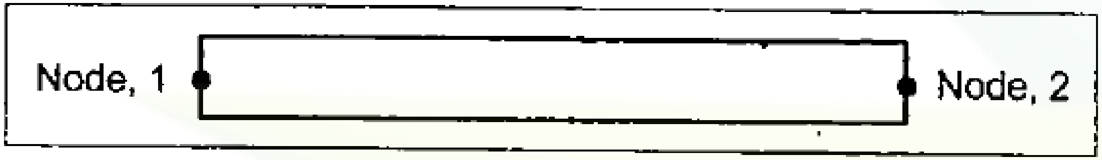
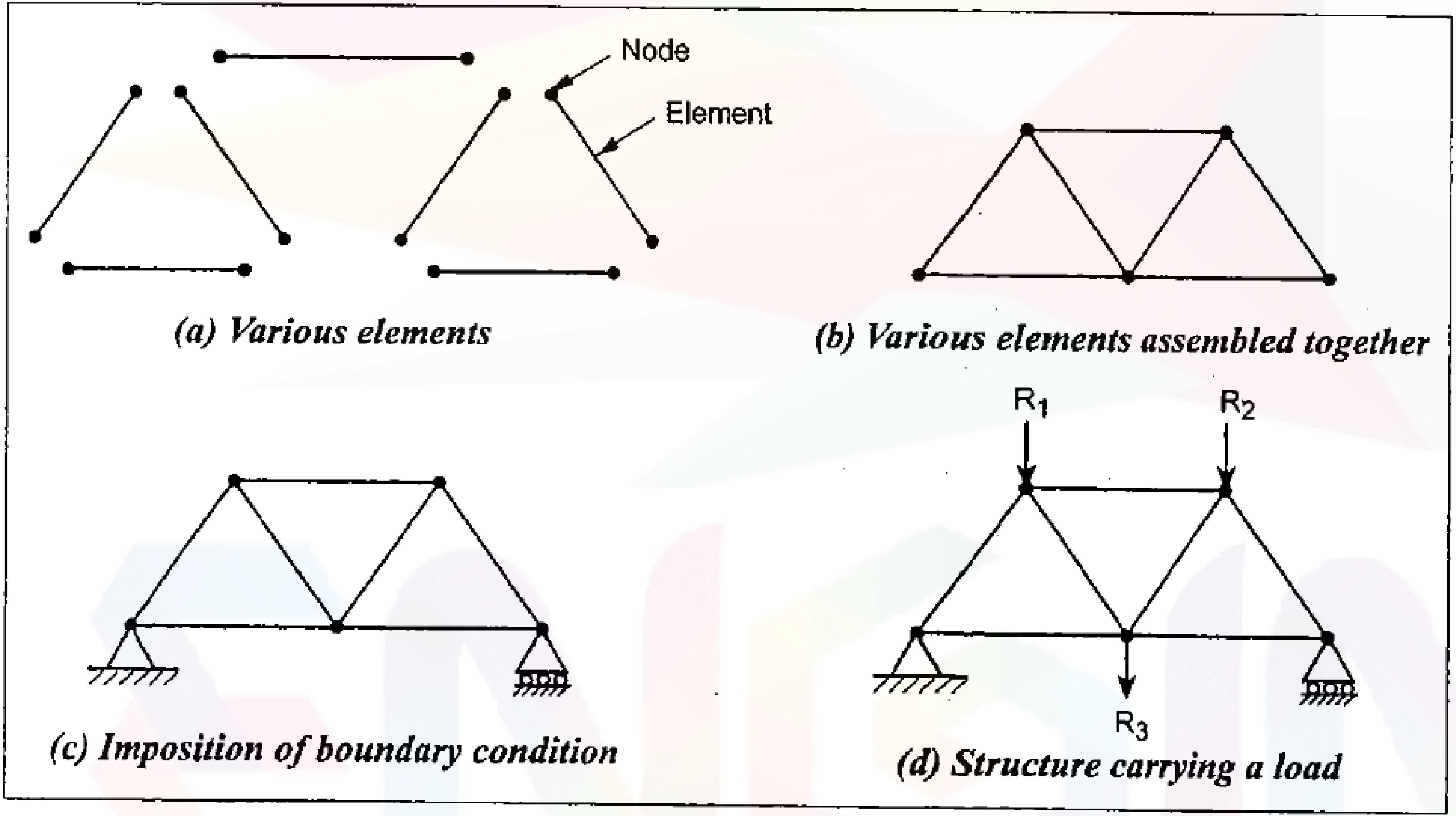


Fig. 1.8. Truss element



(a) Various elements

(b) Various elements assembled together

(c) Imposition of boundary condition

(d) Structure carrying a load

Discretization can be classified as follows:

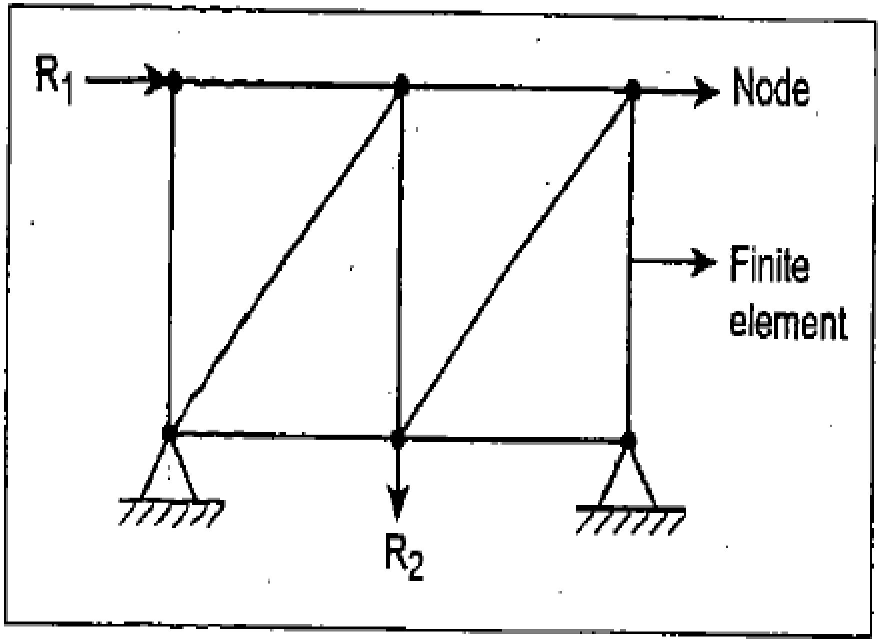
(*i*) Natural.

(*ii*) Artificial (continuum).

Natural Discretization

In structural analysis, a truss is considered as a natural system. The various members of the truss constitute the elements. These elements are connected at various joints known as nodes.

Nodal Points: Each kind of finite element has a specific structural shape and is interconnected with the adjacent elements by nodal points or nodes.



9 elements, 4 freely moving nodes and 2 constrained nodes

Natural discretization of truss

Nodal forces: The forces that act at each nodal point are called nodal forces.

Degrees of freedom: When the force or reaction act at nodal point, node is subjected to deformation. This deformation includes displacements, rotations, and/or strains. These are collectively known as degrees of freedom or simply we can say nodal displacement is called degrees of freedom.

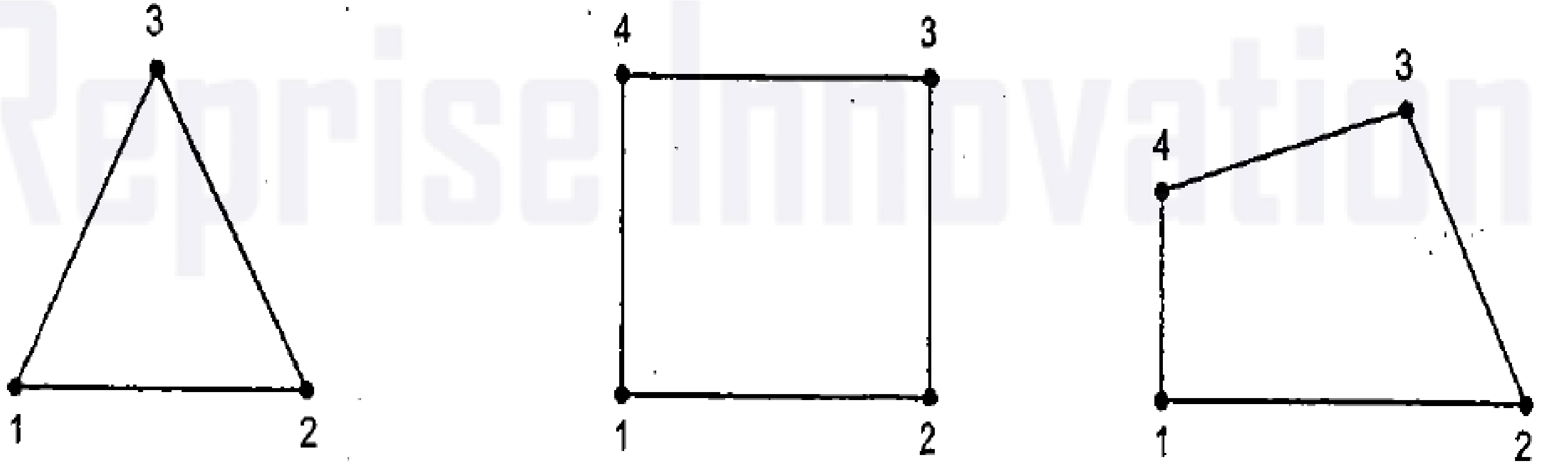
Artificial Discretization (Continuum)

Continuum is generally considered to be a single mass of material as found in a forging, concrete dam, deep beam, plate and so on.

Unlike the truss element which is physically present in the truss, in a continuum, the following three elements exist only in our imagination.

1. Triangular element.
2. Rectangular element.
3. Quadrilateral element.

They are shown in Fig.1.11.



1. Triangular element　*2. Rectangular element*　*3. Quadrilateral element*

Discretization using triangular element is shown in Fig.1.12 & 1.13.

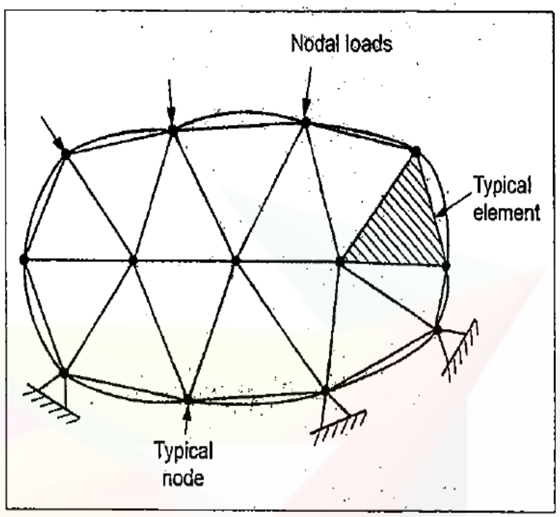


Fig.1.14 shows a deep beam. In Fig.1.15, it is shown how it is discretized using simp rectangular elements.

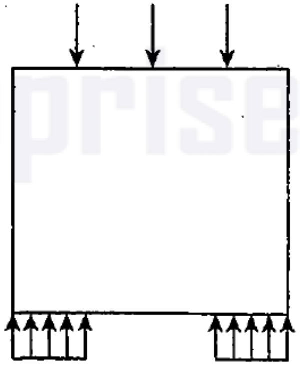


Fig. 1.14. Deep beam

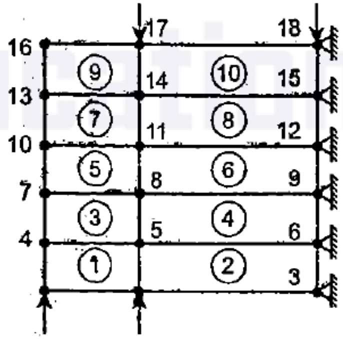
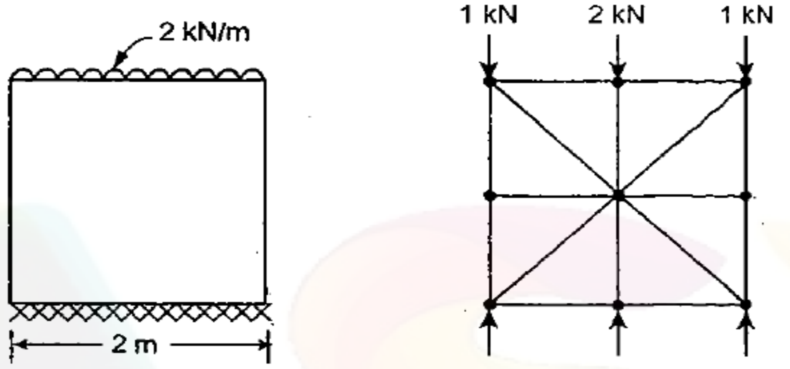
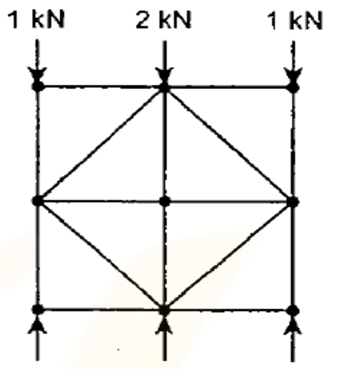


Fig. 1.15. Deep beam discretization using rectangular elements

Fig.1.16 (*b*), the continuum is discretized into eight triangular elements. The discretization shown is only one way. We can subdivide the continuum into triangular elements in a number of ways. Alternative way is shown in Fig.1.17.



(a) Continumm *(b) Discretization*



Discretization process

Following points are to be considered while analyzing the Discretization process.

i. Type of elements

  - Selection of elements depends on no. of degrees of freedom req.
  - Expected accuracy
  - Necessary equation required.

- In some cases structure cant be discretized using single type of element.

ii. Size of elements

  - ➢ if size of element is small, final solution is more accurate. But the computational time is too large.
  - ➢ Aspect ratio should be close to unity.

iii. Location of nodes.

- ✓ If the structure has no abrupt changes in geometric, load, boundary conditions and material properties, the structure can be divided into equal subdivisions. So, the spacing of the nodes are uniform.
- ✓ If there are any discontinuities in geometric, load, boundary conditions and material properties of the structure, nodes should be introduced at these discontinuities as shown in the following figures.

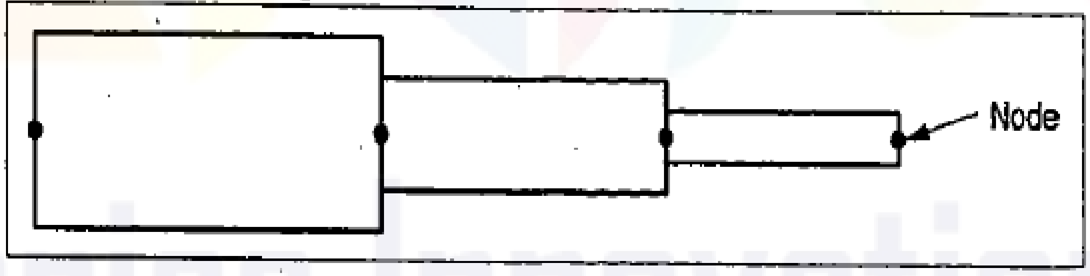


Fig. 1.21. Geometric discontinuities

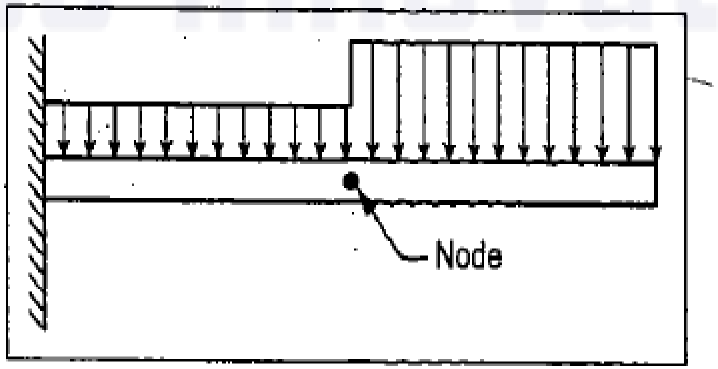


Fig. 1.22. Discontinuity in loading

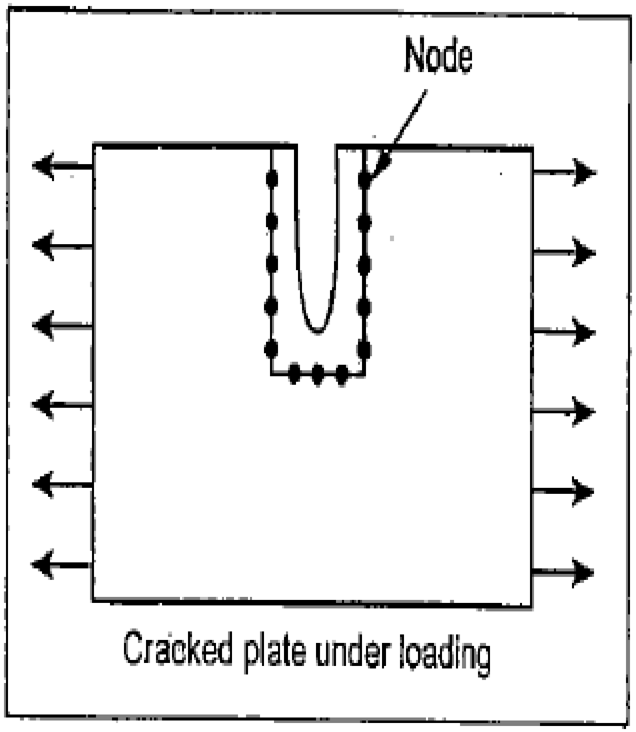


Fig. 1.23. Discontinuity of boundary conditions

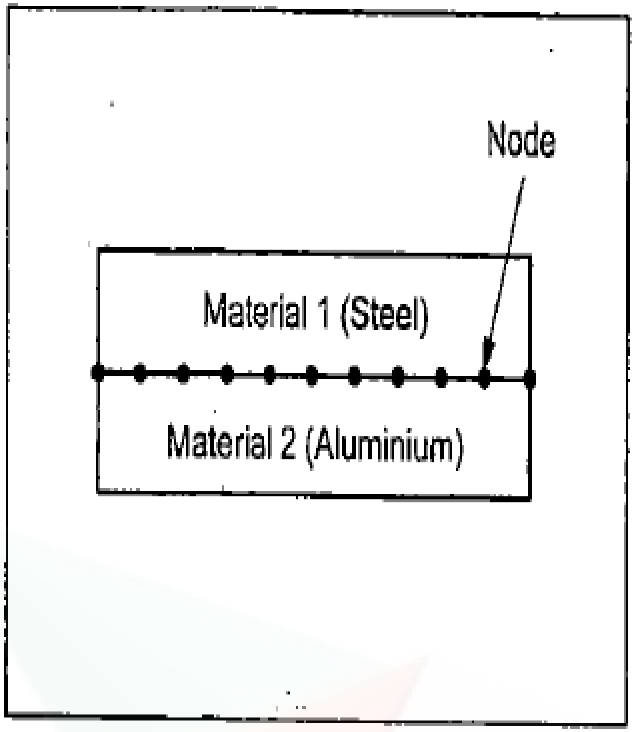
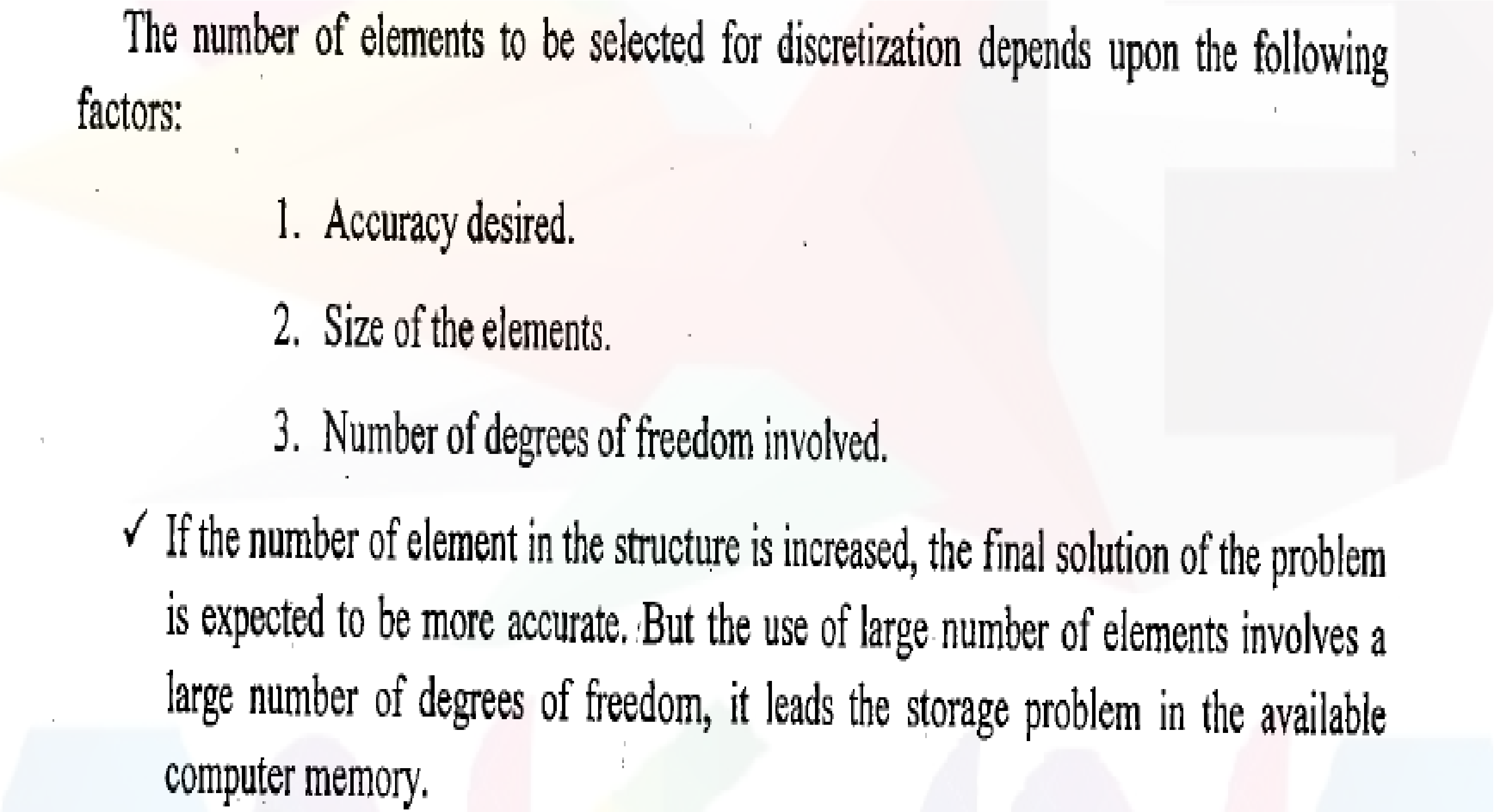


Fig. 1.24. Material discontinuity

iV. No. of elements.



The number of elements to be selected for discretization depends upon the following factors:

1. Accuracy desired.
2. Size of the elements.
3. Number of degrees of freedom involved.

✓ If the number of element in the structure is increased, the final solution of the problem is expected to be more accurate. But the use of large number of elements involves a large number of degrees of freedom, it leads the storage problem in the available computer memory.

Finite Element Modeling

Finite element modelling consists of the following:

(*i*) Discretization of structure.

(*ii*) Numbering of nodes.

(i) Discretization

The art of subdividing a structure into a convenient number of smaller components is known as discretization.

Consider a bar as shown in Fig.2.2. The first step is to model the bar as a stepped shaft. Let us model the bar using 5 finite elements, each having a uniform cross section as shown in Fig.2.3. Every elements connects two nodes. Five element, six node model element is shown in Fig.2.4.

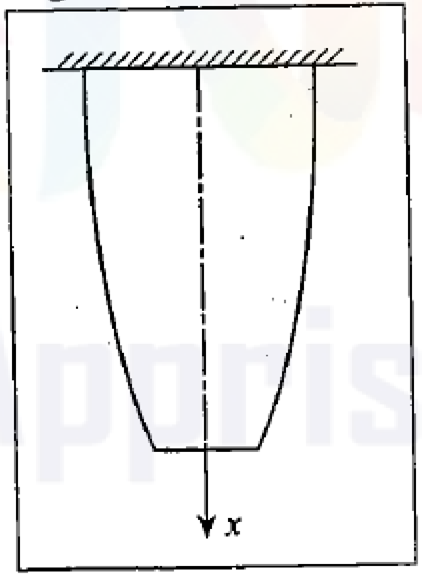


Fig. 2.2.

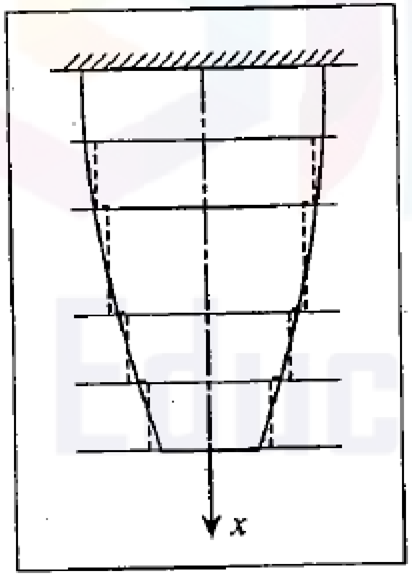


Fig. 2.3.

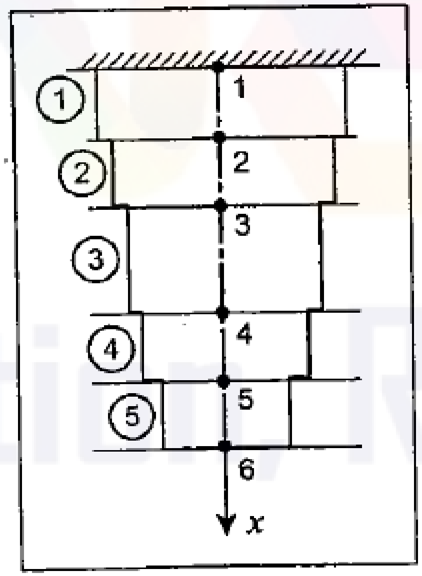


Fig. 2.4.

The element numbers are circled to distinguish them from node numbers. The cross-sectional area, traction forces and body forces are constant within each element. But, these are differ in magnitude from element to element. Better results are obtained by increasing the number of finite elements.

(ii) Numbering of nodes

In one dimensional problem, each node is allowed to move only in ±x direction. So, each node has one degrees of freedom. (Degrees of freedom is nothing but a nodal displacement).

A six node finite element model is shown in Fig.2.5. It has six degrees of freedom. Load is considered as positive if it is acting along the +x direction.

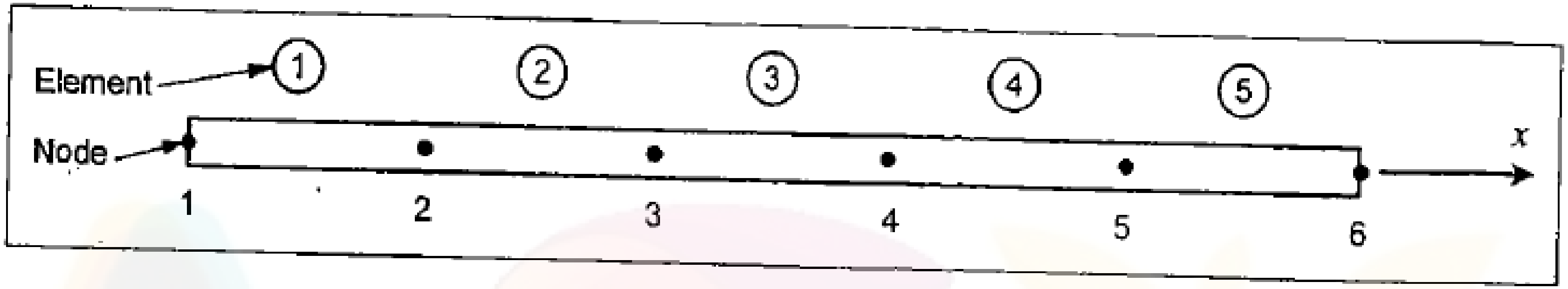
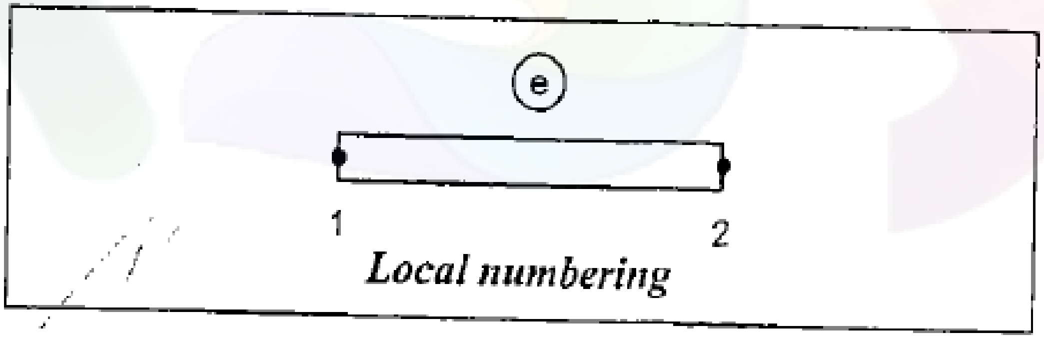
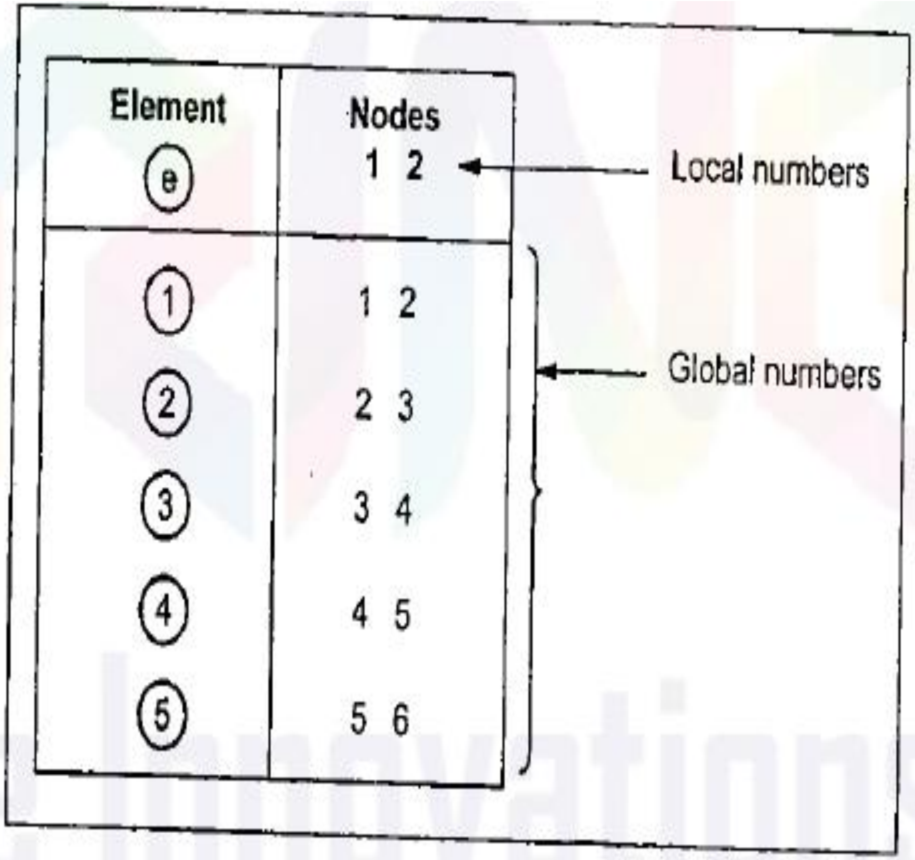


Fig. 2.5.

In the element connectivity table, the heading 1 and 2 refer to local node numbers of an element and the corresponding node numbers on the structure are called global numbers. Connectivity thus establishes the local-global correspondence.



Local numbering



Element ⓔ	Nodes 1 2
①	1 2
②	2 3
③	3 4
④	4 5
⑤	5 6

Fig. 2.6. (b) Connectivity table